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Improved BER and minimized OBI while reducing PAPR by using New Companding Transform

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Abstract— The concept of Orthogonal Frequency Division Multiplexing (OFDM) has been known since 1966. OFDM is an attractive modulation technique for transmitting large amounts of digital data over radio waves. One major disadvantage of OFDM is that the time domain OFDM signal which is a sum of several sinusoids leads to high peak to average power ratio (PAPR). The number of techniques proposed for reducing the PAPR in OFDM systems. These techniques can mainly be categorized in to signal scrambling techniques and signal distortion techniques. The signal distortion techniques introduce both In band (IBI) and Out-of-band (OBI) interference and complexity to the system. The signal distortion techniques reduce high peaks directly by distorting the signal prior to amplification. Those are clipping, peak windowing, Envelop scaling. Companding is also one of the signal distortion technique but which is able to offer an improved bit error rate (BER) and minimized OBI while reducing PAPR effectively then compare to exponential companding. The companding technique can be used to improve OFDM transmission performance. Companding is highly used in speech signal processing where high peaks occur infrequently.

Index Terms— BER,companding, IBI,OBI, OFDM,PAPR,

1 Introduction

OFDM depends on Orthogonality principle. Orthogonality means, it allows the sub carriers, which are orthogonal to each other, meaning that cross talk between co-channels is eliminated and inter-carrier guard bands are not required. This greatly simplifies the design of both the transmitter and receiver, unlike conventional FDM a separate filter for each sub channel is not required. Orthogonal Frequency Division Multiplexing (OFDM) is a digital multi carrier modulation scheme, which uses a large number of closely spaced orthogonal sub-carriers as shown in fig (1).

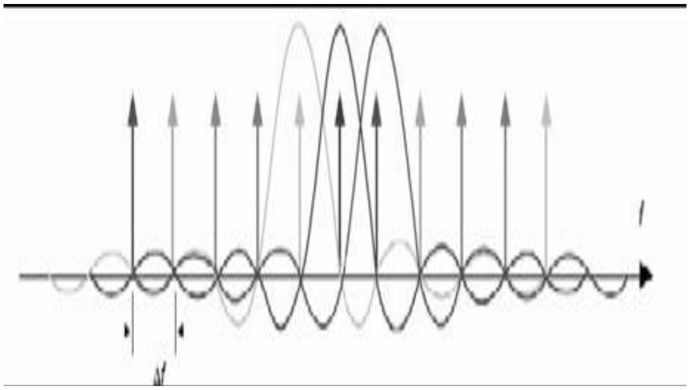
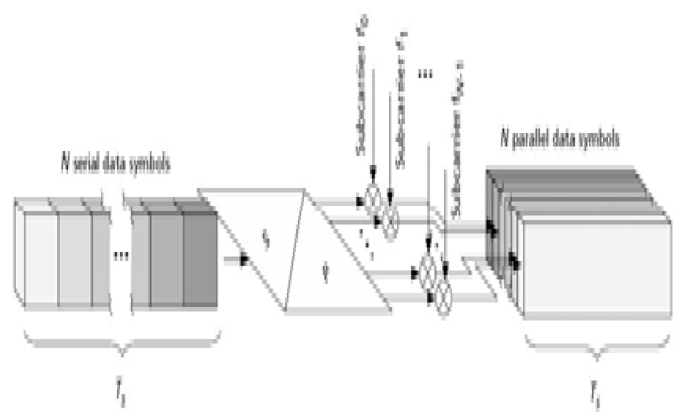


Fig (1)

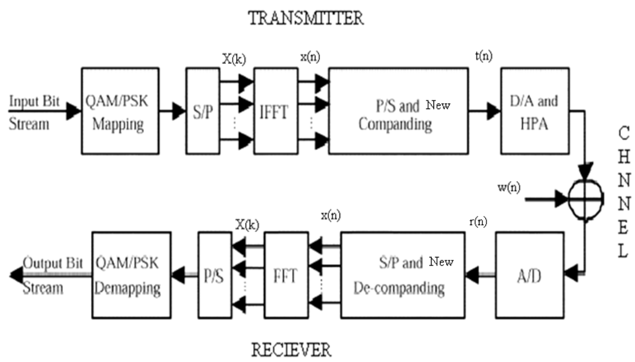
A single stream of data is split into several parallel bit streams each of which is coded and modulated on to a subcarrier as shown in fig(2). Each sub-carrier is modulated with a conventional modulation scheme (such as quadrature amplitude modulation or phase shift keying) at a low symbol rate, maintaining data rates similar to conventional single carrier modulation schemes in the same bandwidth. Thus the high bit rates seen before on a single carrier is reduced to lower bit rates on the subcarrier. In practice, OFDM signals are generated and detected using the Fast Fourier Transform algorithm. OFDM has developed into a popular scheme for wideband digital communication, wireless as well as copper wires.



fig(2)

2 OFDM system model

To generate OFDM successfully the relationship between all the carriers must be carefully Controlled to maintain the orthogonally of the carriers. For this reason, OFDM is generated by firstly choosing the spectrum required based on the input data, and modulation scheme used. Each carrier to be produced is assigned same data to transmit. The required amplitude and phase calculated based on the modulation scheme. The required spectrum is then converted back to its time domain signal using an Inverse Fourier Transform (IFT) as shown fig (3).In most applications, an Inverse Fast Fourier Transform (IFFT) is used. The IFFT performs the transformation very efficiently and provides a simple way of ensuring the carrier signals produced are orthogonal which an input to new compander. The Fast Fourier Transform (FFT) transforms which is a cyclic time domain signal into its equivalent frequency domain spectrum is obtained.



Fig(3)

The amplitude and phase of the sinusoidal components represent the frequency spectrum of the time domain signal. The IFFT performs the reverse process, transforming a spectrum (amplitude and phase of each component) into a time domain signal. An IFFT converts a number of complex data points, of length that is a power of 2, into the time domain signal of the same number of points. Each data point in frequency spectrum used for an FFT or IFFT is called a bin. The orthogonal carrier required for the OFDM signal can be easily generated by setting the amplitude and phase of each frequency bin, thus performing the IFFT.

Problem of peak to average power ratio in OFDM systems

High Peak-to-Average Power Ratio has been recognized as one of the major practical problem involving OFDM modulation. High PAPR results from the nature of the modulation itself where multiple subcarriers are added together to form the signal to be transmitted.

When N sinusoids add, the peak magnitude would have a value of N. High PAPR signals are usually undesirable for it usually strains the analog circuitry (i.e. A/D, D/A, HPA). High PAPR signals would require a large range of dynamic linearity from the analog circuits which usually results in expensive devices and high power consumption with lower efficiency (for e.g. power amplifier has to operate with larger back-off to maintain linearity). In OFDM system, some input sequences would result in higher PAPR than others. For example, an input sequence that requires all such carriers to transmit their maximum amplitudes would certainly result in a high output PAPR. Thus by limiting the possible input sequences to a smallest sub set, it should be possible to obtain output signals with a guaranteed low output PAPR .The PAPR of the transmit signal x(t) is the ratio of the maximum instantaneous power and the average power.

Continuous time PAPR: $$PAPR[x(t)] = \frac{\max[|x(t)|^2]}{E[|x(t)|^2]}$$

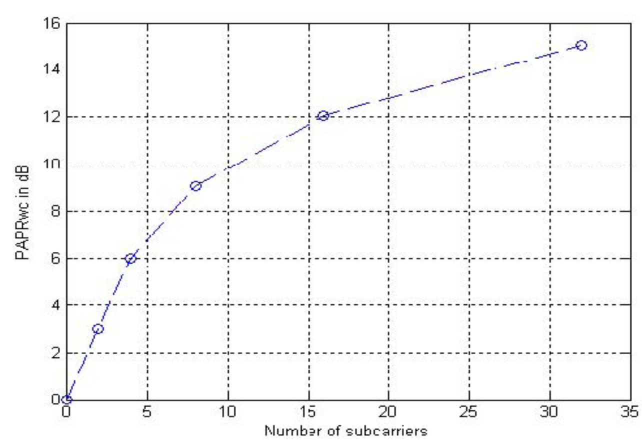
Discrete time PAPR: $$PAPR[x(n)] = \frac{\max[|x(n)|^2]}{E[|x(n)|^2]}$$

If a signal is a sum of N signals each of maximum amplitude equal to 1 Volt, then it is conceivable that we could get maximum amplitude of N Volts, that is, all N signals add at a moment at these maximum points. For an OFDM signal, that has 126 carriers each with normalized power of 1W, then the maximum PAPR can be as large as 10 log10 126 or 21 db. This is at the instant when all 126 carriers combine at their maximum point unlikely but possible. The RMS PAPR will be around half of the number as 10-12 db.

The large amplitude variation increases in-band noise and increases the Bit Error Rate (BER) which the signal has to go through amplification nonlinearities. The crest factor is widely used in the literature as well, which is defined as the square root of the PAPR.

Crest Factor, C.F = $\sqrt{PAPR}$

High PAPR / crest factor could cause problems when the signal is applied to a transmitter which contains non-linear components such as High Power amplifier (HPA) in the Transmitter chain. The PAPR has the worst case value PAPRWC which depends on the no. of subscribers N as shown fig(4). The non-linear effects on the transmitted OFDM symbols are spectral spreading, inter-modulation and changing the signal constellation. In other words, the nonlinear distortion causes both in-band and out-of-band interference to signals. The in band interference increases the Bit Error Rate (BER) of the received signal, while the out-of band interference causes adjacent channel interference through spectral spreading.



Fig(4)

A better solution is to prevent the occurrence of such nonlinear distortion by reducing PAPR of the transmitted signal with some manipulation of the OFDM signal itself some methods like Signal signal distortion techniques

Signal signal distortion techniques

The signal distortion techniques introduce both in band and Out-of-band interference and complexity to the system. The signal distortion techniques reduce high peaks directly by distorting the signal prior to amplification.

1) Clipping and Filtering

In this approach, the amplitude peaks are corrected (or signal is modified) in such a way that a given amplitude threshold of the signal is not exceeded after the correction. The OFDM signal is corrected by adding it with a corrective function k(t). This correction limits the signal s(t) to A0 at positions of amplitude peaks. This method produces no out-of-band interference and causes interference of the OFDM signal with minimal power. If the OFDM signal is not oversampled, then the correction scheme is identical with clipping and each correction of an amplitude peak causes interference on each sub carrier and the power of the correcting function is distributed evenly to all sub carriers. To apply this correcting scheme, the signal s(t) is oversampled by a factor of four and normalized so that the signal power is one. Then the signal is corrected with k(t). For the correction the amplitude threshold A0 is set according to the input bakeoff. After the correction, the signal is limited to the amplitude A0 in order to take into account the limitation of amplitude peaks which may have remained. The signal can be corrected by multiplicative Gaussian function or additive sinc function. The interesting part of the scheme is that it can be used for any number of subcarriers and it does not need any redundancy. The PAPR is reduced at the cost of small increase in the total in band distortion.

2) Peak Windowing

Peak Windowing technique provides better PAPR reduction with better spectral properties than clipping. Peak windowing can achieve PAPR around 4dB for an arbitrary subcarriers, at the cost of slight increase in BER and out-of-band (OOB) interference. In windowing technique a large signal peak is multiplied with a certain window, such as Gaussian shaped window, cosine, Kaiser and Hamming window. Since the OFDM signal is multiplied with several of these windows, the resulting spectrum is a convolution of the original OFDM spectrum with the spectrum of the applied window. Ideally the window should be as narrow band as possible, on the other hand the window should not be too long in the time domain because that implies that many signal samples are affected increasing the BER. With windowing method, PAPR can be reduced down to about 4dB, independent of the number of sub carriers.

3) Envelope Scaling

This is an algorithm to reduce PAPR by scaling the input envelope for some sub carriers before they are sent to IFFT. The main idea behind the scheme is that the envelopes of all the subcarriers input, with PSK modulation, are equal. The envelope of the input in some subcarriers can be scaled to obtain the minimum PAPR at the output of IFFT. The final input that gives the lowest PAPR will be sent to the system. The input sequences have the same phase information as the original one but the envelopes are different.

So the receiver can decode the received sequence without any side information. The main idea behind the scheme is that the envelope of the input in some subcarriers is scaled to obtain the minimum PAPR at the output of the IFFT.

4) Companding

Companding is also one of the signal distortion technique but which able to offer an improved bit error rate (BER) and minimized OBI while reducing PAPR effectively. The companding technique can be used to improve OFDM transmission performance. The law of companding technique is used to compand the OFDM signal before it is converted into analog waveform. The OFDM signal, after taking IFFT, is companded and quantized. After D/A conversion, the signal is transmitted through the channel. At the receiver end hen the received signal is first converted into digital form and expanded. Companding is highly used in speech processing where high peaks occur infrequently. Companding technique improves the quantization resolution of small signals at the price of the reduction of the resolution of large signals, since small signals occur more frequently than large ones.

Due to companding, the quantization error for large signals is significantly large which degrades the BER performance of the system. So the companding technique improves the PAPR in expense of BER performance of the system.

a) Exponentail Companding

A new nonlinear companding technique, namely "exponential companding", that can effectively reduce the PAPR of transmitted (companded) OFDM signals by transforming the statistics of the amplitudes of these signals into uniform distribution. The new scheme also has the advantage of maintaining a constant average power level in the nonlinear companding operation. The strict linearity requirements on HPA can then be partially relieved.

Let $|t(n)|^d$,the d^{th} power of the amplitude of companded signal t(n) ,have a uniform distribution in the interval [0,α].The exponent d is called the degree of a specific exponential companding scheme.

Where the companding function

$$h(x) = sign(x(n)).\sqrt[d]{\alpha.\left[1-\exp(-\frac{|x(n)|^2}{\sigma^2})\right]}$$

Where sign(x(n)) is the sign function.The positive constant α determines the average power of output signals.In order to keep the input and output signals at the same average power level.

Where $$\alpha = \left(\frac{E[|x(n)|^2]}{E\sqrt[d]{\left[1-\exp(-\left(\frac{|x(n)|^2}{\sigma^2}\right)\right]}}\right)^{\frac{d}{2}}$$

At the receiver side, the inverse function of h(x) is used in the decompanding function

$$h^{-1}(x) = sign(x(n)).\sqrt{-\sigma^2 \log_e\left[1-\frac{x(n)^d}{\alpha}\right]}$$

b) New companding

Here we proposed a new companding algorithm using asmooth function, namely the special function, which is second order differential equation (y^{11}-xy=0).

The solution for the equation is $\frac{1}{\Pi}\int_0^{\infty}\cos\left(\frac{1}{3}t^3 + xt\right)dt$

The companding function is as follows

$$f(x) = t(n) = \beta.sign(x(n))[airy(0) - airy(\alpha.|x(n)|)]$$

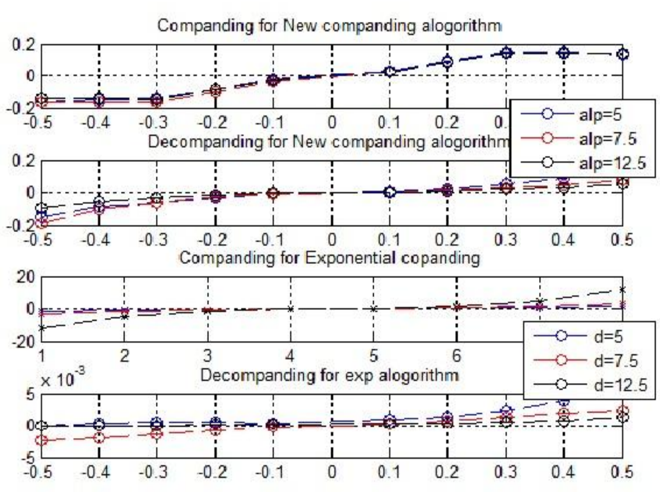
Where airy (.) is the airy function of the first kind.Where x(n) is the input samples to the compander. Where α is the parameter that controls the degree of companding. β is the factor adjusting the average output power of the compander to the same level as the average input power

Where $$\beta = \sqrt{\frac{E[|x(n)|^2]}{E[|airy(0) - airy(\alpha.|x(n)|)|^2]}}$$

Where E[.] denotes the expectation.
The decompanding function is the inverse of the f(x)

Where $f^{-1}(x) = \frac{1}{\alpha}.sign(x(n)).airy^{-1}\left[airy(0) - \frac{|x(n)|}{\beta}\right]$

Notice that the input the compander is a quantized signal with finite set of values.Therefore numerically pre-compute $f^{-1}(x)$.The simulated results for New companded & decompanded and Exponential companded & decompanded as shown in fig(5)



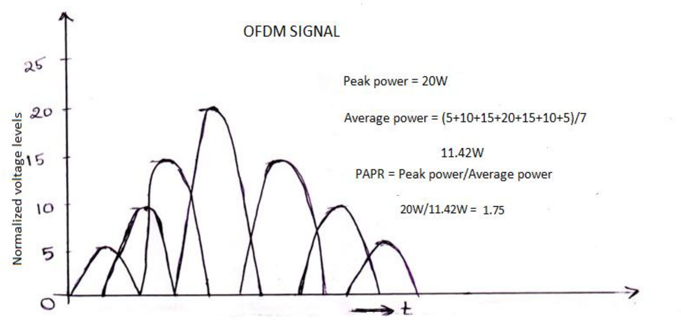
Fig(5)

PAPR performance algorithm

High Peak-to-Average Power Ratio has been recognized as one of the major practical problem involving OFDM modulation.

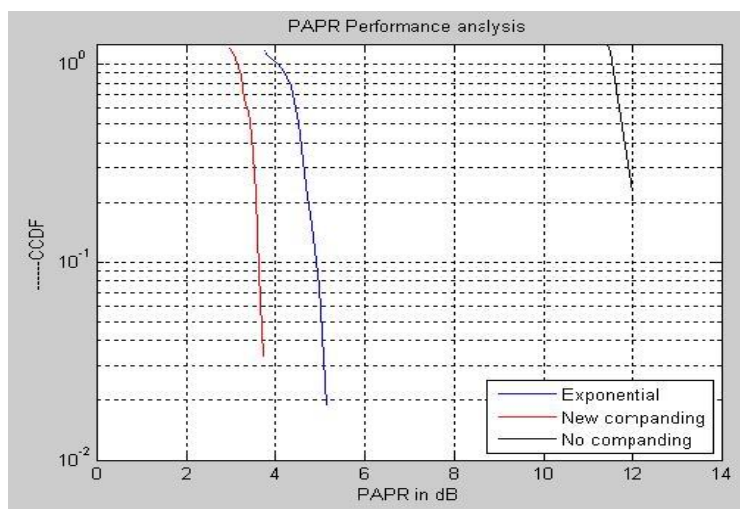
High PAPR results from the nature of the modulation itself where multiple subcarriers are added together to form the signal to be transmitted.High PAPR signals required a large range of dynamic linearity from the analog circuits which usually results in expensive devices and high power consumption with lower efficiency. In OFDM system,some input sequences results in higher PAPR than others. For example, an input sequences that requires all such carriers to transmit their maximum amplitudes would certainly result in a high output PAPR.

Thus by limiting the possible input sequences to a smallest sub set, it should be possible to obtain output signals with a guaranteed low output PAPR.The PAPRdefined as PAPR of the transmit signal x(t) is the ratio of the maximum instantaneous power and the average power. For a mathematical example exmple as shown in fig(6).



Fig(6)

CCDF (Complimentary cumulative distribution function) vs peak ti average power ratio of the two companding schemes as shown in fig(7).



Fig(7)

BER performance algorithm

we examine the BER performance of the algorithm hera.

Let y(n) denote the output signal of the compander, where w(n) are the samples of AWGN signal w(t) .

The received signal can be expressed as f(x(n))=y(n)=r(n)=t(n)+w(n)
After decompanding operation

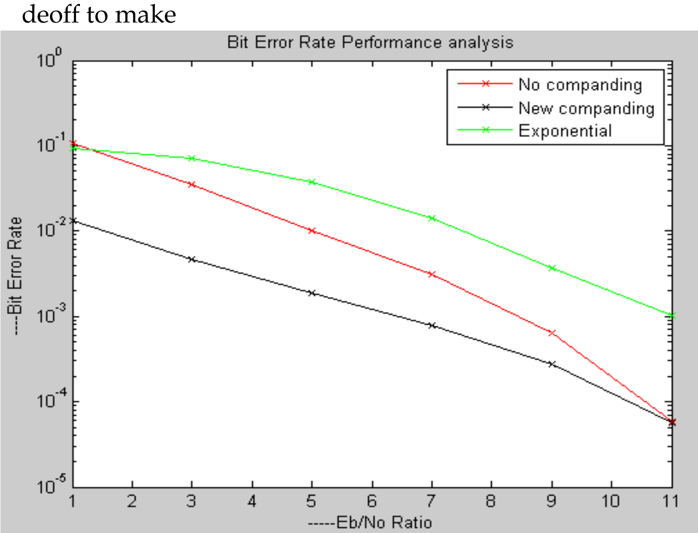
$r'(n) = f^{-1}(r(n)) = t(n) + f^{-1}(w(n))$

Using the first order Taylor series expansion, can be approximated

$$\bar{r}(n) = t(n) + \frac{df^{-1}(u)}{du}\Big|_{u=y(n)} . w(n)$$

From the given Equation shows that if y(n) falls into the range of the decompanding function f−1(u) where df−1(u)/du|u=y(n) < 1, the noise w(n) is suppressed, and if y(n) is out of the range, df−1(u)/du|u=y(t) > 1 and the noise is enhanced. Therefore, if the parameter α is properly chosen such that more y(n) is within the noise-suppression range of f−1(u), it is possible to achieve better overall BER performance. Where transmitted signal t(n) and received signal $r'(n)$ are compared for calculating bit error rate. The simulated results for bit error rate vs signal to noise ratio is as shown in fig(8).

Notice that the signal-to-noise ratio (SNR) in a typical additive white Gaussian noise (AWGN) Channel is much greater than 1. It is worth to mention though that BER and PAPR affect each other adversely and therefore there is a tradeoff to make



Fig(8)

OBI performance algorithm

Out band interference is the spectral leakage into alien channels. To know Quantification of the OBI caused by companding requires the knowledge of the power spectral density (PSD) of the companded signal. Unfortunately PSD is in general mathematically intractable for analytical expression, because of the nonlinear companding transform involved.

To estimate OBI here we take an alternative approach. Let us consider f(x) be a nonlinear companding function, and x(t) = sin(ωt) be the input to the compander.

The companded signal y (t) = f[x(t)] = f[sin(ωt)] Since y(t) is a periodic function with the same period as x(t), y(t) can then be expanded into the following Fourier series:

$$y(t) = \sum_{k=-\infty}^{+\infty} c(k)e^{jk\omega t},$$

Where the coefficients c(k) is calculated as:

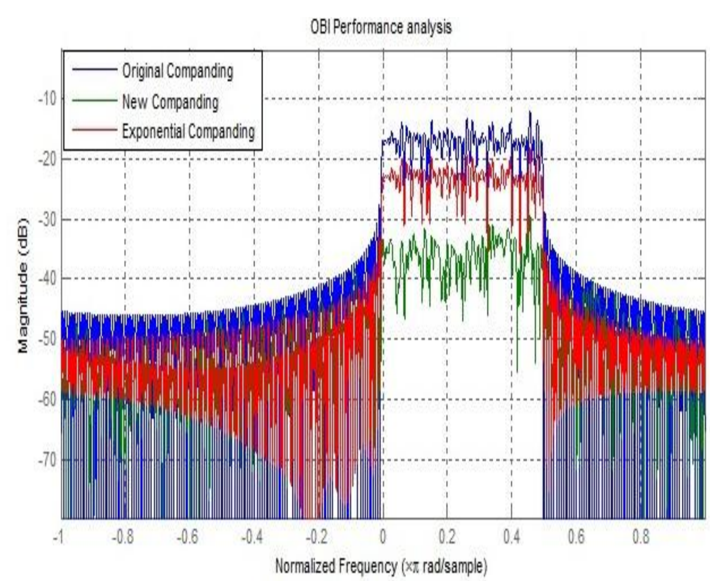
$$c(k) = c(-k) = \frac{1}{T}\int_0^T y(t)e^{-jk\omega t}dt \qquad T = \frac{2\pi}{\omega}.$$

Notice that the input x in this case is a pure sinusoidal signal, any c(k) ≠0 for |k| > 1 is the OBI produced by the nonlinear companding process. Therefore, to minimize the OBI, c(k) must approach to zero fast enough as k increases. It has been shown that $c(k) \cdot k^{-(m+1)}$ tends to zero if y(t) and its derivative up to the m-th order, or in other words, c(k) converges at the rate of $k^{-(m+1)}$. Given an arbitrary number n, the n-th order derivative of y(t), $d^n y/dt^n$, is a function of $d^i f(x)/dx^i$, (i = 1, 2, · · · ,n), as well as sin(ωt) and cos(ωt), i.e.:

$$c(k) = \frac{d^n y}{dt^n} = g\left(\frac{d^n f(x)}{dx^n}, \frac{d^{n-1} f(x)}{dx^{n-1}}, \cdots, \frac{df(x)}{dx}, \sin(\omega t), \cos(\omega t)\right)$$

Where sin(ωt) and and cos(ωt) are continuous functions, $d^n y/dt^n$ is continuous if and only if the condition satisfied $d^i f(x)/dxi$ (i = 1, 2, · · · ,n) are continuous.

Based on this observation we can conclude, Companding introduces minimum amount of OBI if the companding function f(x) is infinitely differentiable. The functions that meet the above condition are the smooth functions. We now propose a new companding algorithm using a smooth function, namely the airy special function.



Fig(9)

Performance simulation analysis

The OFDM system used in the simulation consists of 64 QPSK-modulated data points. The size of the FFT/IFFT is 256, meaning a 4× oversampling. Given the compander input power of 3dBm, the parameter α in the companding function is chosen to be 30. Consequently about 19.6 percent of y *(t)* is within the noise-suppression range of the decompanding function.

One of the popular companding algorithm, namely an exponential companding, is also included in the simulation for the purpose of performance comparison.

CCDF (Complimentary cumulative distribution function) of the two companding schemes as shown in fig(). The new companding algorithm is roughly 3.52dB than compare to the exponential (4.951dB) and its original signal.

The BER vs. SNR is plotted in Fig(7).Our new companding algorithm outperforms the other one.The bit error rate for new companding(0.0007792) is very less than copare to an exponential companding(0.0136).So improved OFDM system performance.

The simulated PSD(Power Spectral Density) of the companded signals is illustrated in Fig(6). The new companding algorithm produces OBI almost 14dB lower than the exponential algorithm, 21dB lower than the original signal.

One more observation from the simulation is unlike the exponential companding whose performance is found almost unchanged under different degrees of companding; the new companding algorithm is flexible in adjusting its specifications simply by changing the value of *α* in the companding function.

Conclusion

So, we have proposed a new companding transformation. Both theoretical analysis and computer simulation show that the algorithm offers improved performance in terms of BER and OBI while reducing PAPR effectively.

Acknowledgement

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